

Exercise 73

The water level, measured in feet above mean sea level, of Lake Lanier in Georgia, USA, during 2012 can be modeled by the function

$$L(t) = 0.01441t^3 - 0.4177t^2 + 2.703t + 1060.1$$

where t is measured in months since January 1, 2012. Estimate when the water level was highest during 2012.

Solution

The domain of the function is $0 \leq t \leq 11$. Take the derivative.

$$\begin{aligned} L'(t) &= \frac{d}{dt}(0.01441t^3 - 0.4177t^2 + 2.703t + 1060.1) \\ &= 0.01441(3t^2) - 0.4177(2t) + 2.703(1) + 1060.1(0) \\ &= 0.04323t^2 - 0.8354t + 2.703 \end{aligned}$$

Set $L'(t) = 0$ and solve for t .

$$0.04323t^2 - 0.8354t + 2.703 = 0$$

$$t = \frac{0.8354 \pm \sqrt{0.8354^2 - 4(0.04323)(2.703)}}{2(0.04323)}$$

$$t = 15.2151 \text{ months} \quad \text{or} \quad t = 4.10948 \text{ months}$$

$t = 4.10948$ is within the interval $0 \leq t \leq 11$, so evaluate the function here.

$$\begin{aligned} L(4.10948) &= 0.01441(4.10948)^3 - 0.4177(4.10948)^2 + 2.703(4.10948) + 1060.1 \\ &\approx 1065.15 \text{ ft} \quad (\text{absolute maximum}) \end{aligned}$$

Evaluate the function at the endpoints.

$$L(0) = 0.01441(0)^3 - 0.4177(0)^2 + 2.703(0) + 1060.1 = 1060.1 \text{ ft}$$

$$L(11) = 0.01441(11)^3 - 0.4177(11)^2 + 2.703(11) + 1060.1 \approx 1058.47 \text{ ft} \quad (\text{absolute minimum})$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $0 \leq t \leq 11$. Therefore, the water level was highest 4.10948 months after January 1, 2012, which was on May 3, 2012.

The graph below illustrates these results.

